<u>Adding Fractions in Which the Denominators are Not the Same by Finding the Lowest Possible Common Denominator:</u> When adding fractions with unlike denominators in which the larger/largest of the denominators you are working with can not be used as the common denominator, the denominators of both fractions must be changed:

<u>Example</u>:

$$\begin{array}{r} \frac{7}{10} \\ + \; \frac{1}{4} \\ \hline \end{array}$$

<u>Step #1</u>: Find the lowest common denominator for the fractions $\frac{7}{10}$ and $\frac{1}{4}$. Remember that a common denominator can be found by multiplying both original denominators together (4 x 10 = 40) as was shown in section #2 of this study-guide. In addition, the larger of the two denominators can be used as the common denominator only if the smaller of the two can be divided evenly into it. However, since **4** can not be divided evenly into **10**, you can not use **10** as the common denominator. Instead, you should focus on the larger of the two denominators with which you are working (in this case, the number 10), and identify all the multiples of **10** until you find one that can also be evenly divided by the smaller of the two original denominators (4).

(a) Find the multiples of **10**: 10 x **1** = **10**; 10 x **2** = **20**; 10 x **3** = **30**; 10 x **4** = **40**; and so on.

(b) Ask yourself, "What is the lowest multiple of 10 that can also be divided evenly by **4**?" In this case, it is the number **20**.

<u>**Step #2**</u>: Change the original denominators of both fractions being added to the common denominator (20):

$$\frac{7}{10 \times 2} = \frac{}{20}$$

$$\frac{1}{4 \times 5} = \frac{}{20}$$

<u>**Step #3**</u>: Change each of the numerators of the original fractions to new numerators by multiplying the **original numerators** times the same numbers you used to multiply the original denominators to create the common denominator:

(**a**) In the case of the fraction $\frac{7}{10}$, the original denominator **10** was multiplied times the number **2** in order to create the common denominator **20**. As a result, the original numerator of the fraction (7) must also be multiplied times the number **2** in order to create the new numerator, **14** (2 x 7 = 14):

$$\begin{array}{rccccl} \text{original numerator} & \underline{7} & \times & 2 & = & \underline{14} \ \textbf{new numerator} \\ \text{original denominator} & 10 & \times & 2 & = & 20 \ \textbf{common denominator} \end{array}$$

(b) In the case of the fraction $\frac{1}{4}$, the original denominator (4) was multiplied times the number **5** to create the common denominator **20**. As a result, the original numerator of the fraction (1) must also be multiplied times the number **5** in order to create the new numerator, **5** (5 x 1 = 5):

$$\begin{array}{rccccl} \text{original numerator} & \underline{1} & \times & 5 & = & \underline{5} \ \textbf{new numerator} \\ \text{original denominator} & 4 & \times & 5 & = & 20 \ \textbf{common denominator} \end{array}$$

Step #4: Once the denominators of the original fractions have been changed to the common denominator and the numerators have been changed accordingly, the problem can be solved as shown:

$$\begin{array}{r} \frac{14}{20} \\ + \ \frac{5}{20} \\ \hline \frac{19}{20} \end{array}$$

Step #5: Check to make sure that the answer is expressed in simplest form. Since **19** and **20** have no other common factor other than the number **1**, the original answer is in simplest form, $\frac{19}{20}$.

Adding Whole Numbers and Fractions: When adding a fraction to a whole number, simply bring the whole number directly down in the whole number position in the answer and write the fraction being added to the right side of the whole number.

Example:

$$\begin{array}{r} 19 \\ + \ \frac{1}{4} \\ \hline 19\frac{1}{4} \end{array}$$

Adding Mixed Numerals When the Denominators of the Fractions are the Same: When both denominators of the fraction parts of the mixed numerals are the same, you can simply add the numerators of the fractions together and keep the denominator the same, just as is done with adding proper fractions. Next, add the whole numbers of each mixed numeral together to get the final answer. **Note:** Remember that a mixed numeral is a whole number combined with a fraction:

Example #1:

$$\begin{array}{r} 4\frac{2}{5} \\ + \ 2\frac{1}{5} \\ \hline 6\frac{3}{5} \end{array}$$

Note: Once you have solved the problem, make sure the fraction part of the answer is in simplest form. In this case, $\frac{3}{5}$ is in the simplest form, so the answer remains $6\frac{3}{5}$.

Example #2:

$$\begin{array}{r} 3\,\frac{1}{6} \\ +\ 2\,\frac{2}{6} \\ \hline 5\,\frac{3}{6} \end{array} \quad \begin{array}{c} \div\ 3 \\ \div\ 3 \end{array} \ = \ 5\,\frac{1}{2}$$

Note: In Example #2, the fraction part of the mixed numeral in the answer ($\frac{3}{6}$) can be reduced by dividing both the numerator and denominator by the common factor **3.**

Note: Once you have reduced the fraction part of the mixed numeral to simplest form, the final answer is $5\,\frac{1}{2}$.

Example #3:

$$\begin{array}{r} 4\,\frac{7}{8} \\ +\ 2\,\frac{5}{8} \\ \hline 6\,\frac{12}{8} \end{array}$$

Note: In this case, the fraction part of the answer $\frac{12}{8}$ is an improper fraction and must be changed to a mixed numeral (see "Introduction to Fractions" study-guide for an explanation of how to change improper fractions to mixed numerals).

Step #1: Change the improper fraction $\frac{12}{8}$ to the mixed numeral. $1\frac{4}{8}$

Step #2: Reduce the fraction part of the mixed numeral ($\frac{4}{8}$) to simplest form by using the common factor **4**:

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

Step #3: Add the new mixed numeral $1\frac{1}{2}$ you created when you changed $\frac{12}{8}$ to $1\frac{1}{2}$ to the whole number part of the answer (6) to get the final answer:

$$6 + 1\frac{1}{2} = 7\frac{1}{2}$$

<u>Adding Mixed Numerals When the Fractions Have Different Denominators</u>: When adding mixed numerals in which the fraction parts of each number have different denominators, you must first change each fraction to a common denominator before you can work the problem.

<u>Example</u>:

$$\begin{array}{r} 3\,\frac{1}{4} \\ +\ 2\,\frac{2}{5} \\ \hline \end{array}$$

<u>Step #1</u>: Change the denominators and numerators of the original fractions so that both fractions have the same denominator. The lowest possible common denominator for the two original fraction parts of the mixed numerals being added is **20.**

$$3\,\frac{1 \times 5}{4 \times 5} = \frac{5}{20} \quad \& \quad 2\,\frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

<u>Step #2</u>: Add the mixed numerals after the fraction parts of each mixed numeral have been converted to the common denominator and the numerators have been changed accordingly:

$$\begin{array}{r} 3\,\frac{5}{20} \\ +\ 2\,\frac{8}{20} \\ \hline 5\,\frac{13}{20} \end{array}$$

<u>Step #3</u>: Make sure the fraction part of the mixed numeral in the answer is in simplest form. In this case, $\frac{13}{20}$ is simplest form, so the final answer remains $5\frac{13}{20}$.

<u>Adding More Than Two Fractions or Mixed Numerals at a Time</u>: The rules for adding more than two fractions or mixed numerals at a time are the same for adding only two fractions or mixed numerals (See next page).

<u>Example #1</u>: Adding more than two fractions when all fractions have the same denominator.

$$\frac{1}{7} + \frac{2}{7} + \frac{3}{7}$$

<u>Step #1</u>: Check to see that all fractions being added have the same denominators. In this example, all three fractions being added have a common denominator (7). Therefore, the problem can be solved immediately by adding the numerators and using the original common denominator in the answer:

$$\begin{array}{r} \frac{1}{7} \\ \frac{3}{7} \\ +\ \frac{2}{7} \\ \hline \frac{6}{7} \end{array}$$

Step #2: Check to make sure that the answer is in simplest form and can not be reduced. Since there are no common factors for 6 and 7 other than **1**, the answer is in simplest form and should remain as $\frac{6}{7}$.

Example #2: Adding more than two proper fractions when the denominators are not the same.

$$\begin{array}{r} \frac{1}{3} \\ \frac{1}{4} \\ + \frac{1}{5} \\ \hline \frac{6}{7} \end{array}$$

Step #1: Change each fraction being added to a common denominator, and change the numerators as was demonstrated in previous examples. In this case, the number **60** is the lowest possible common denominator for the three fractions being added.

$$\frac{1 \times 20}{3 \times 20} = \frac{20}{60}$$

$$\frac{1 \times 15}{4 \times 15} = \frac{15}{60}$$

$$\frac{1 \times 12}{5 \times 12} = \frac{12}{60}$$

Step #2: After each fraction has been changed to the common denominator, and the numerators have been changed accordingly, add the fractions:

$$\begin{array}{r} \frac{20}{60} \\ \frac{15}{60} \\ + \frac{12}{60} \\ \hline \frac{47}{60} \end{array}$$

Step #3: Check to make sure the answer is expressed in simplest form. Since there are no common factors for 47 and 60 other than **1**, the answer is in simplest form and remains $\frac{47}{60}$.

Example #3: Adding more than two mixed numerals in which the fractions have the same denominator:

$$\begin{array}{r} 4\frac{1}{8} \\ 2\frac{3}{8} \\ + \ 1\frac{1}{8} \\ \hline \end{array}$$

Step #1: Check to see that the fraction parts of all the mixed numerals being added have a common denominator. In this example, all the fraction sections of each original mixed numeral have the same denominator (8). Therefore, no changes need to be made before solving the problem:

$$\begin{array}{r} 4\frac{1}{8} \\ 2\frac{3}{8} \\ +1\frac{1}{8} \\ \hline 7\frac{5}{8} \end{array}$$

Step #2: Check to make sure the answer is expressed in simplest form: In this case, the fraction part of the answer $\left(\frac{5}{8}\right)$ can not be reduced. Therefore, the answer remains $\mathbf{7\frac{5}{8}}$.

Example #4: Adding mixed numerals in which the fraction parts of each mixed numeral are not the same:

$$\begin{array}{r} 2\frac{1}{4} \\ 11\frac{1}{2} \\ +12\frac{2}{3} \\ \hline \end{array}$$

Step #1: Change each fraction part of the mixed numerals being added to a common denominator and change the numerators accordingly. In this case, the number **12** is used as the common denominator:

(a) $2\frac{1 \times 3}{4 \times 3} = 2\frac{3}{12}$

(b) $11\frac{1 \times 6}{2 \times 6} = 11\frac{6}{12}$

(c) $12\frac{2 \times 4}{3 \times 4} = 12\frac{8}{12}$

Step #2: After changing each fraction to the common denominator (12) and changing the numerators accordingly, add the mixed numerals together:

$$\begin{array}{r} 2\frac{3}{12} \\ 11\frac{6}{12} \\ +12\frac{8}{12} \\ \hline 25\frac{17}{12} \end{array}$$

Step #3: Check to make sure the answer is expressed in simplest form. Since the fraction part of the answer is an improper fraction, $\left(\frac{17}{12}\right)$, it must be changed to a mixed numeral:

$$\begin{array}{r} 1 \\ 12\overline{)17} \\ \underline{12} \\ 5 \end{array} = 1\frac{5}{12}$$

Step #4: Add the mixed numeral you created in step #3 $\left(1\frac{5}{12}\right)$ to the sum of the original whole numbers $(2+11+12=25)$:

$$\begin{array}{r} 25 \\ +\ 1\frac{5}{12} \\ \hline 26\frac{5}{12} \end{array}$$

Step #5: Check to make sure the final answer is expressed in simplest form. The fraction part of the final answer $\left(\frac{5}{12}\right)$ can not be reduced. Therefore, the answer remains $\mathbf{26\frac{5}{12}}$.

Subtracting Fractions With The Same Denominators: When the denominators of the fractions being subtracted are the same, simply subtract the numerators and keep the same denominator in the answer, just as is done when adding fractions that have the same denominator:

$$\begin{array}{r} \frac{2}{8} \\ -\ \frac{1}{8} \\ \hline \frac{1}{8} \end{array}$$

Note: Make sure that the answer is expressed in simplest form. In this case, the original answer $\left(\frac{1}{8}\right)$ can not be reduced, so the answer is already in simplest form.

Subtracting Fractions When the Denominators Are Not the Same: When subtracting fractions in which the denominators are not the same, use the same rules and steps as are used in adding fractions with unlike denominators.

Example:

$$\begin{array}{r} \frac{1}{2} \\ -\ \frac{1}{3} \\ \hline \end{array}$$

Step #1: First you must find a "common denominator" for the two fractions you are subtracting. To do this, follow the same procedures you used to find a common denominator when adding fractions with different denominators. In the case of the problem in this example, the lowest possible common denominator is the number **6**.

After changing each original denominator to the new common denominator, change the original numerators of the fractions being subtracted in the same way as was shown in earlier examples:

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Step #2: Once both fractions have been changed so that they have a common denominator, subtract the numerators, and keep the same denominator in the answer:

$$\begin{array}{r} \frac{3}{6} \\ -\ \frac{2}{6} \\ \hline \frac{1}{6} \end{array}$$

Step #3: Reduce the answer to simplest form. In this case, the original answer can not be simplified. Therefore, $\mathbf{\frac{1}{6}}$ is the final answer.

Subtracting a Fraction From a Mixed Numeral When the Denominators of the Fraction Parts of Both Numbers Are the Same: As long as the denominators of the fractions are the same, and the numerator of the fraction in the mixed numeral is larger than the one being subtracted from it, you can perform the operation immediately.

Example:

$$\begin{array}{r} 7\frac{5}{9} \\ -\ \frac{3}{9} \\ \hline 7\frac{2}{9} \end{array}$$

Note: The fraction part of the answer can not be reduced, so the final answer is $\mathbf{7\frac{2}{9}}$

Subtracting One Mixed Numeral From Another When the Fraction Parts of Both Mixed Numbers Are the Same: As long as the denominators of both fraction parts of the mixed numerals being subtracted are the same and the numerator of the fraction part of the mixed numeral is smaller than the one from which it is being subtracted, you can perform the operation immediately.

Example:

$$\begin{array}{r} 4\frac{9}{11} \\ -\ 3\frac{7}{11} \\ \hline 1\frac{2}{11} \end{array}$$

Note: The fraction part of the answer ($\frac{2}{11}$) can not be reduced. So, the final answer remains $\mathbf{1\frac{2}{11}}$

Subtracting a Fraction From a Whole Number: When subtracting a fraction from a whole number, you must first borrow from the whole number and create a mixed numeral before you can perform the operation. To do this, you must create a fraction that has the same denominator as the fraction being subtracted. In order for the new fraction to be equal to the whole number you are borrowing, the numerator must be the same number as the denominator (see "Introduction to Fractions" page **7** of this study-guide for further explanation about how to change a whole number to fraction form).

Example:

$$\begin{array}{r} 11 \\ -\ \frac{1}{3} \\ \hline \end{array}$$

Step #1: Borrow from the whole number and create a fraction next to it. Since the denominator of the fraction being subtracted from the whole number is **3**, the numerator and the denominator of the fraction you create when you borrow must also be **3** in order to equal the whole number you borrowed:

$$\begin{array}{r} \overset{10}{\cancel{11}}\ \frac{3}{3} \\ -\ \frac{1}{3} \\ \hline \end{array}$$

Step #2: Once the borrowing process has been completed, you can perform the subtraction operation:

$$\begin{array}{r} 10\ \frac{3}{3} \\ -\ \frac{1}{3} \\ \hline 10\ \frac{2}{3} \end{array}$$

Step #3: Check to make sure the fraction part of the mixed numeral in the answer is in the simplest form. In this case, the fraction part of the answer $\left(\frac{2}{3}\right)$ is in simplest form, so the answer remains $\mathbf{10\frac{2}{3}}$.

Subtracting One Mixed Numeral From Another When You Must Borrow From the Whole Number of the Mixed Numeral and Add to the Fraction Part: If the numerator of the fraction part of the mixed numeral being subtracted is larger than the one from which it is being subtracted, you must borrow from the whole number (in the top mixed numeral) before you can perform the operation. Follow the same basic procedure used for subtracting a fraction from a whole number explained in section #13.

Notice that you will create an improper fraction when you borrow from the whole number and add it to the fraction part of the mixed numeral.

Example:

$$\begin{array}{r} 5\frac{3}{11} \\ -\ 3\frac{7}{11} \\ \hline \end{array}$$

Step #1: Borrow from the whole number of the top mixed numeral and add to the fraction part.

$$\begin{array}{rl} \overset{4}{\cancel{5}}\,\frac{3}{11} & + \ \frac{11}{11} \\ -\ 3\,\frac{7}{11} & \\ \hline \end{array}$$

Step #2: After you have borrowed from the whole number in the mixed numeral from which you are subtracting, and added to the fraction part, you can perform the operation:

$$\begin{array}{r} 4\frac{14}{11} \\ -\ 3\frac{7}{11} \\ \hline 1\frac{7}{11} \end{array}$$

Step #3: The fraction part of the answer ($\frac{7}{11}$) can not be reduced. So, the final answer remains $\mathbf{1\frac{7}{11}}$

Subtracting One Mixed Numeral From Another When You Must Convert the Fraction Parts of Each Mixed Numeral to a Common Denominator and Borrow From the Whole Number of the Mixed Numeral From Which you Are Subtracting: When the denominators of both mixed numerals being subtracted are not the same, you must change the fraction part of each mixed number to a common denominator. After both denominators and numerators have been changed, if the numerator of the fraction in the mixed numeral being subtracted from is smaller than the numerator of the fraction part of the mixed number being subtracted from it, you must borrow from the whole number in the top mixed numeral to add to the fraction part before you can solve the problem.

Example:

$$\begin{array}{r} 12\frac{1}{5} \\ -\ 8\frac{3}{4} \\ \hline \end{array}$$

Step #1: First, you must change the fractions in each mixed numeral so that both have a common denominator. In this case, the number **20** is used as the common denominator. Therefore, the numerators of the fraction parts of both mixed numerals must be changed accordingly:

$$12\ \frac{1 \times 4}{5 \times 4} = \frac{4}{20} \qquad \& \qquad 8\ \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

Step #2: Next, because the fraction part of the first mixed numeral $\left(\frac{4}{20}\right)$ is smaller than the one being subtracted from it $\left(\frac{15}{20}\right)$, you must borrow from the whole number of the top mixed numeral to add to the fraction part:

$$\begin{array}{r} \overset{11}{\cancel{12}}\,\frac{4}{20} + \frac{20}{20} = 11\,\frac{24}{20} \\ -\;8\,\frac{15}{20} \qquad\qquad -\;8\,\frac{15}{20} \\ \hline \end{array}$$

Step #3: Once the fraction parts of both mixed numerals have been changed to the common denominator, and the borrowing procedure has been completed with the top mixed numeral, you can solve the problem:

$$\begin{array}{r} 11\,\frac{24}{20} \\ -\;8\,\frac{15}{20} \\ \hline 3\,\frac{9}{20} \end{array}$$

Step #4: Check to make sure the fraction part of the mixed numeral in the answer is in simplest form. In this case, $\frac{9}{20}$ is in simplest form, so the final answer remains $\mathbf{3\frac{9}{20}}$.